

Distributed Multi-Relay Selection in Accumulate-then-Forward Energy Harvesting Relay Networks

Yifan Gu, He Chen, Yonghui Li, and Branka Vucetic

Abstract

This paper investigates a wireless-powered cooperative network (WPCN) consisting of one source-destination pair and multiple decode-and-forward relays. Contrary to the conventional cooperative networks, we consider the scenario that all relays have no embedded energy supply, but they are equipped with energy harvesting units and rechargeable batteries. The relays are thus able to harvest and accumulate the energy from source's signal, and use the accumulated energy to forward source's information. We develop an energy threshold based multi-relay selection (ETMRS) scheme for the considered WPCN. We then derive an approximate closed-form expression for the system outage probability of the proposed ETMRS protocol over mixed Nakagami-m and Rayleigh fading channels. A heuristic approach is proposed to optimize the energy threshold for each relay to avoid the high computation complexity of the exhaustive search approach. To gain some useful insights into the practical design, we derive the upper bound for system outage probability corresponding to the case that all relays are equipped with infinite-capacity batteries. Numerical results validate our theoretical analysis, and show that the proposed heuristic approach can achieve a near-optimal system performance, and our ETMRS scheme outperforms the existing single-relay selection scheme, especially when the source's transmission rate is relatively high.

The material in this paper was presented in part at the IEEE International Conference on Communications, Kuala Lumpur, Malaysia, in May 2016.

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Index Terms

Wireless energy harvesting, cooperative communications, relay selection, accumulate-then-forward, Markov Chain, outage probability.

I. INTRODUCTION

The performance of many wireless communication networks in practice is largely confined by the energy constrained devices that require replenishment periodically. Recently, a novel radio-frequency (RF) energy transfer and harvesting technique has been proposed as a new viable and promising solution to prolong the lifetime of energy constrained wireless networks [1]. With RF energy transfer and harvesting, wireless devices are enabled to harvest energy from RF signals broadcast by ambient/dedicated energy transmitters to charge their batteries [2], [3]. This technique has opened a new research paradigm, termed wireless-powered communication (WPC), which has become a hot research topic recently (see, e.g., [4], [5] and references therein).

The WPC technique has brought new research opportunities to cooperative communications, which have attracted an upsurge of interest during the past decade due to its various advantages [6]. Particular, the relay node is now able to harvest energy from ambient RF signals and then use the harvested energy to assist the source node's transmission. In this sense, the relay is more willing to cooperate with the source since it does not need to consume its own energy. In this paper, we refer to a cooperative communication network with wireless-powered¹ relay(s) as a wireless-powered cooperative network (WPCN). In [7], Nasir *et. al* first investigated a classical three-node WPCN consisting of one source-destination pair and one energy harvesting amplify-and-forward (AF) relay. Two practical relaying protocols, namely time switching-based relaying and power splitting-based relaying, were proposed and analyzed in [7]. Inspired by this seminal work, a plenty of works focusing on the design and/or analysis of WPCNs have published in open literature very recently (see [8]–[22] and references therein).

All aforementioned existing works on WPCNs considered that the wireless-powered node(s) splits the received signals into two parts in time, power, antenna or space domain: one part is used for energy harvesting and the other part is used for information processing. This is motivated by the fact that the up-to-date circuit technology still cannot support harvesting energy

¹Throughout this paper, we use the terms “wireless-powered” and “energy harvesting” interchangeably.

and retrieving information from the same RF signal. Moreover, for simplicity, it is assumed that the wireless-powered node(s) exhausts the harvested energy in the current time slot to perform information transmission/forwarding straight away. However, this assumption may have some practical issues and lead to only a suboptimal system performance. Specifically, when the receiving channel of the wireless-powered node suffers from deep fading, it can only harvest a little amount of energy and thus it may not be able to perform an effective information transmission/forwarding even it exhausts all the harvested energy. On the other hand, when the transmitting channel of the wireless-powered node is in good condition, it should use part of its harvested energy for information transmission/forwarding and save the rest energy for future use. In this sense, it is essential to equip each wireless-powered node with an energy storage (e.g., a rechargeable battery) such that they can accumulate the harvested energy and then perform information transmission/forwarding in an appropriate time slot. However, to the best knowledge of authors, only a few papers in open literature incorporated the energy accumulation process in the design and analysis of WPCNs. The energy accumulation process of the classical three-node WPCNs with a single wireless-powered relay was modeled and the resulting network performance was analyzed in [23], [24] for finite and infinite storage scenario, respectively.

In practice, multiple relays with energy accumulation capability may co-exist between a source-destination pair. Motivated by this, Liu extended [23] into a multi-relay WPCN in [25], where multiple energy harvesting relays are deployed to assist information transmission between source and destination. Inspired by the opportunistic relaying (OR) originally proposed in [26], a battery-aware relay selection (BARS) scheme was proposed and analyzed. In the BARS, the relays with accumulated energy exceeding a predetermined threshold will first form a subset, which feeds back their channel state information (CSI) to the source. Then, the “best” relay among the subset with maximum end-to-end signal-to-noise ratio (SNR) is selected by the source to forward its information, while other relays harvest energy in the first hop. The outage probability of the BARS scheme was characterized and it was concluded that the diversity gain of BARS scheme heavily depends on the number of potential relays and the overhead for collecting required information for relay selection. Note that the BARS scheme proposed in [25] only chooses single relay. This may limit the system performance since the harvested energy at relays are generally limited, especially when the source’s transmit power is low. In this case, several relays with limited energy could work cooperatively and create a virtual multiple antenna array to avoid

such outage.

Very recently in [27], the authors studied a WPCN with multiple randomly distributed relays. To make the performance analysis tractable, the wireless-powered relays are assumed to be active only when they are fully charged. Based on this assumption, the authors proposed five relay selection schemes, including random scheme, distance based scheme, energy based scheme, distance and energy based scheme and the distributed beamforming (DB) scheme. In the DB scheme, all relay nodes which are full at the beginning of the transmission block form a forwarding subset. Among this subset of relays, the relays that are able to decode the source's signal create a virtual multiple antenna array and transmit source's signal to destination coherently. In the DB scheme proposed in [27], full CSI of all relays falling into the forwarding subset is required to perform coherent transmission in the second hop. This indicates that the relays in the forwarding subset need to exchange information before information forwarding, which not only consumes their limited amount of harvested energy but also increases network latency. Furthermore, the relays are active only when they are fully charged, which is not optimal, particularly for those relays with good channel conditions in the second hop.

Motivated by the above issues, in this paper we develop an energy threshold based multi-relay selection (ETMRS) scheme with energy accumulation capability at each relay for WPCNs. In the proposed ETMRS, each relay can flexibly switch between energy harvesting and information forwarding modes in a certain transmission block based only on its own battery status and its local CSI of the second hop. We consider a practical and more general channel fading than [25], [27], where source-relay links and relay-destination links are assumed to experience *independent but not necessarily identical distributed* channel fading. Besides, different from [27], we adopt a multiple-level battery model to characterize the charging/discharging behavior at relay batteries. In this case, the amount of harvested energy available at relays could be different. As such, to exploit the relays adequately, in the ETMRS scheme we set an individual energy threshold for each relay. The performance of the proposed ETMRS scheme can be improved by jointly optimizing the energy thresholds of all relays. This is in contrast to the DB scheme in [27], where the energy thresholds of different relays are the same and equal to the battery capacity. Moreover, compared with the DB scheme in which the weights in distributed beamforming for the relays within the forwarding set are obtained based on full CSI of the whole forwarding subset, the weight at each relay in the proposed ETMRS approach is calculated based only on the local CSI

such that it can be implemented in a fully distributed manner. This can effectively reduce the network overhead and latency. Due to the aforementioned practical considerations, performance analysis methods of the existing schemes (e.g., [8]–[25], [27]) is not applicable to characterize the system performance of the proposed ETMRS scheme and a new approach should be developed. Furthermore, the performance characterization of the ETMRS scheme is not straightforward at all by realizing that the end-to-end SNR depends on the dynamic charging/discharging behaviors of all relays and involves the sum of independent but not necessarily identical distributed random variables due to the adopted channel fading model.

The main contributions of this paper are summarized as follows:

- We propose an energy threshold based multi-relay selection (ETMRS) scheme for WPCNs, in which each wireless-powered relay determines to switch between energy harvesting and information forwarding modes based only on local battery status and channel information. Thus, the proposed ETMRS can be implemented in a fully distributed manner, which is particularly appealing for the considered system since it would be challenging for energy harvesting relays to perform selection algorithms involving complicated computation and inter-node coordination.
- We consider a practical finite-capacity battery model. To characterize the dynamic charging/discharging behaviors of relays' batteries, we follow [23], [25] and adopt a finite-state Markov Chain (MC) approach. Thanks to the distributed feature of the proposed ETMRS scheme, we are able to calculate the stationary distribution of each relay battery separately.
- Based on the calculated stationary distributions of relays' batteries, we then derive approximate closed-form expression on system outage probability for the proposed scheme over mixed Nakagami-m and Rayleigh fading channels. In our analysis, we use the Nakagami-m fading to model the channel fading of source-relay links because it is mathematically more tractable than the Rician fading and can well approximate the latter at the same time. This is motivated by the fact that the up-to-date wireless energy transfer techniques could only be operated within a relatively short communication ranges such that the line-of-sight (LoS) path is very likely to exist in these links. In this sense, the Rayleigh fading used in [25], [27] to model the source-relay links is somewhat impractical.
- Due to the complexity of the adopted MC model, it is difficult to find a closed-form solution to the optimal energy thresholds for all relays that lead to the minimum system

outage probability. But, based on the derived closed-form expression of the system outage, the optimal energy thresholds can be obtained by applying the exhaustive search-based method. On the other hand, the computation complexity of the method becomes prohibitive as the number of relays and their battery energy levels increase. Motivated by this, we propose a heuristic approach to design the energy threshold for each relay by only using its channel statistical information in both hops. Numerical results demonstrate that this heuristic approach can achieve the near-optimal performance.

- To gain some useful insights for practical relay design, we also derive the upper bound for system outage probability corresponding to the case that all relays are equipped with infinite-capacity batteries. Moreover, numerical results are provided to demonstrate the impacts of several system parameters on the network performance and the merits of the proposed scheme over the BARS² one proposed in [25].

The rest of the paper is organized as follows. Sec. II introduces the system model and elaborates the principles of the proposed ETMRS scheme. The resulting end-to-end SNR is also characterized. In Sec. III, we evaluate the stationary distribution of each relay battery for the proposed scheme using the MC model. In Sec. IV, the approximate closed-form expression of outage probability for the proposed ETMRS scheme are derived. A heuristic approach to optimize the performance of the ETMRS scheme is also provided. The upper bound for system outage probability corresponding to the case that all relays are equipped with infinite-capacity batteries are provided in Sec. V. Numerical results are presented in Sec. VI to validate the analysis and compare the proposed schemes with the existing scheme in open literature. Finally, the conclusions of this paper are drawn in Sec. VII.

Notation: Throughout this paper, we use $f_X(x)$ and $F_X(x)$ to denote the probability density function (PDF) and cumulative distribution function (CDF) of a random variable X . $\Gamma(\cdot)$ is the Gamma function [28, Eq. (8.310)], $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function [28, Eq. (8.350.1)] and $\lceil \cdot \rceil$ is the ceiling function. We use $(\cdot)^*$ and $(\cdot)^T$ to represent the complex conjugate and the transpose of a matrix or vector, respectively. $\mathbb{E}[\cdot]$ is the expectation operator and \mathbf{I} denotes the identity matrix. $P\{A|B\}$ is the conditional probability of A under a given

²The comparison of ETMRS scheme and DB scheme [27] is not provided in this paper by realizing that it is not fair to compare them since the adopted battery model are totally different.

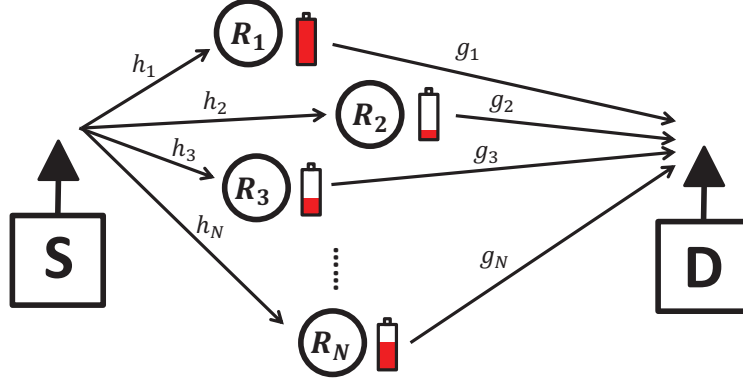


Fig. 1. The considered WPCN with one source-destination pair and N wireless-powered relays.

condition B .

II. SYSTEM MODEL AND SCHEME DESIGN

A. System Model

As depicted in Fig. 1, in this paper we investigate a WPCN consisting of one source-destination pair and N decode-and-forward (DF) relays, which are deployed to assist the source's information transmission. We assume that there is no direct link between source and destination due to obstacles or severe attenuation. Also, all nodes are equipped with single antenna and work in half-duplex mode. As in [27], we consider the scenario that all DF relays are wireless-powered devices and purely rely on the energy harvested from RF signals broadcast by source to perform information forwarding. Moreover, these relays are equipped with separate energy and information receivers [29]. As such, they can flexibly switch between energy harvesting (EH) mode and information forwarding (IF) mode at the beginning of each transmission block. We also assume that each relay is equipped with a finite-capacity rechargeable battery such that it can perform energy accumulation and scheduling across different transmission blocks. Specifically, they can accumulate the harvested energy to a certain amount before assisting the source's information transmission.

We denote by T the duration of each transmission block, which is further divided into two time slots with equal length $T/2$. During the first time slot, the source broadcasts its signal to all relays. At each relay operating in EH mode, the received signal is delivered to the energy receiver to convert to direct current and charge the battery. In contrast, the received signal at



Fig. 2. Time diagram of EH and IF modes.

each relay in IF mode is connected to its information receiver to decode the information sent by source. All relays that operate in IF mode and decode the source's information correctly forms a *decoding set*. In the second time slot, all relays falling in the decoding set will jointly forward the source's information to destination by consuming part of the accumulated energy from their batteries. On the other hand, the relays out of the decoding set keep in silence during the second time slot³. The time diagrams of EH and IF operations are shown in Fig. 2.

We hereafter use subscript- S and subscript- D to denote the source and destination respectively. We denote by R_u , $u \in \{1, 2, \dots, N\}$, the u -th wireless-powered relay. Among existing fading models, Rician fading would be the most appropriate one to characterize the channel fading of S - R_u links. This is mainly motivated by the fact that the up-to-date wireless energy transfer techniques could only be operated within a relatively short communication range such that the line-of-sight (LoS) path is very likely to exist in these links. However, the statistical functions (e.g., cumulative density function (CDF) and probability density function (PDF)) of Rician fading are very complicated, which would make the analysis extremely difficult [30]. Fortunately, the Rician distribution could be well approximated by the more tractable Nakagami- m fading model. Thus, in this paper we adopt an asymmetric scenario for the fading distributions of source-to-relay links and relay-to-destination links. Specifically, the S - R_u link is assumed to be subject to Nakagami- m fading with fading severity parameter m_u and average power gain λ_{SR_u} , while the R_u - D link suffers from Rayleigh fading with average power gain λ_{R_uD} as the distance between them may be much further. Besides, all channels in the considered system experience slow, independent, and frequency-flat fading such that instantaneous channel gains remain unchanged

³For simplicity, we consider that the relays out of the decoding set will not harvest energy in the second time slot since the amount of energy harvested from the forwarded signals would be negligible compared with the energy harvested from the source.

within each transmission block but change independently from one block to the other. It is worth mentioning that we do not require the channel fading parameters to be identical or non-identical for both hops. That is, we investigate a general *independent but not necessarily identical* fading model, which includes the independent and identical one as well as the independent and non-identical one for special cases. Without loss of generality, we consider a normalized transmission block (i.e., $T = 1$) hereafter.

Let P denote the source transmit power and x denote the transmitted symbol with $\mathbb{E}[|x|^2] = 1$. The received signal at the u -th relay during the first time slot is thus given by

$$y_u = \sqrt{P}h_u x + n, \quad (1)$$

where h_u is the channel coefficient between S and R_u , and n denotes the additive white Gaussian noise (AWGN) with zero mean and variance N_0 at the receiver side.

When the u -th relay works in EH mode, the received signal y_u will be delivered to its energy receiver and converted to direct current to charge the battery. The amount of harvested energy at R_u during the first time slot can thus be expressed as

$$\tilde{E}_u = \frac{1}{2}\eta P H_u, \quad (2)$$

where $0 < \eta < 1$ is the energy conversion efficiency and $H_u = |h_u|^2$ is the channel power gain between S and R_u . Note that in (2), we ignore the amount of energy harvested from the noise since the noise power is normally very small and below the sensitivity of the energy receiver. On the other hand, if R_u opts to decode information in the first time slot, it will harvest zero energy.

Let Φ denote the current decoding set. In the second time slot, all relays in the decoding set Φ will jointly forward the source's information to the destination by implementing the distributed beamforming technique [31]. Specifically, the transmitted signal at $R_u \in \Phi$ is given by

$$x_u = w_u \sqrt{P_u} x, \quad (3)$$

where w_u is the weight of R_u in distributed beamforming and P_u is the transmit power of R_u . In the considered WPCN, we assume that each relay only knows its local CSI of the second hop. In this case, the optimal weight for the u -th relay that maximizes the overall end-to-end SNR can be expressed as $w_u = g_u^*/|g_u|$ [31], where g_u is the complex channel coefficient between R_u - D . It is worth pointing out here that the weight for the u -th relay in [27] is designed as $g_u^*/\sqrt{\sum_{i \in \Phi} |g_i|^2}$,

which would require the full CSI of all relays in the decoding set Φ . The adopted distributed beamforming scheme in this paper does not require any inter-relay information exchange and thus is particularly suitable for the considered WPCN, in which the relays are energy harvesting nodes such that it would be challenging for them to perform complicated inter-node coordination for joint transmission.

We define $\hat{g}_u = |g_u|$ for notation simplicity. The received signal at the destination can thus be expressed as

$$y_d = \sum_{u: R_u \in \Phi} \hat{g}_u \sqrt{P_u} x + n. \quad (4)$$

As a result, the conditional end-to-end SNR for a given decoding set Φ can be written as

$$\gamma_\Phi = \frac{\left(\sum_{u: R_u \in \Phi} \sqrt{P_u} \hat{g}_u \right)^2}{N_0}. \quad (5)$$

B. Energy Threshold Based Multi-Relay Selection

Now, several natural yet important questions arises in the considered system. First of all, *which relay(s) should be selected to perform IF, while others perform EH in each transmission block?* This is actually a very complex question to answer. The reason is that if the relays perform IF too often, they may not be able to accumulate enough energy to support an effective information forwarding in the second hop even they successfully decode received signal in the first hop. On the other hand, too frequent EH at relays could also lead to extra system outages as the number of relays falling in the decoding set would be very small. Secondly, *how much energy should each relay in the decoding set consume to forward the information?* Note that this value could be varied for different relays as the amount of accumulated energy available in their batteries may be different. Thirdly, *is it possible to develop a low-complexity and decentralized relay selection scheme such that the relays can determine their own operation mode locally?* This is particularly crucial to the considered system as it would be challenging for the energy harvesting relays to perform selection algorithms involving complicated computation and inter-node coordination.

Motivated by these important questions, in this paper we develop an energy threshold based multi-relay selection (ETMRS) framework for the considered WPCN. In our ETMRS scheme, each relay R_u is equipped with an individual energy threshold, denoted by β_u . Each relay decides to operate in EH mode or IF mode based on its own battery status at the beginning

of each transmission block. Specifically, relay R_u will perform the IF operation only when its accumulated energy is not less than its associated energy threshold β_u . Otherwise, it will opt EH mode to further accumulate energy in its battery. Moreover, if R_u works in the IF mode and falls in the decoding set, it will forward the signal to destination by consuming the amount of energy β_u from its battery in the second time slot. The conditional end-to-end SNR given in (5) can now be updated by substituting $P_u = \beta_u/(1/2) = 2\beta_u$. That is,

$$\gamma_\Phi = \frac{\left(\sum_{u: R_u \in \Phi} \sqrt{2\beta_u} \hat{g}_u \right)^2}{N_0}. \quad (6)$$

It is worth pointing out here that the energy thresholds $\beta_u, u = 1, 2, \dots, N$, for all the relays should be optimized jointly in order to achieve the best system performance. When the relays are in different locations, the optimal energy thresholds could be distinct due to the fact that the average amount of harvested energy at each relay highly depends on its location. Moreover, for the special case that the relays are co-located in a cluster, the channels are now independent and identically distributed (i.i.d.). Obviously, the relays should now adopt an identical energy threshold, denoted by $\beta = \beta_1 = \beta_2 = \dots = \beta_N$. We then have the conditional end-to-end SNR of this i.i.d. case given by

$$\hat{\gamma}_\Phi = \frac{2\beta \left(\sum_{u: R_u \in \Phi} \hat{g}_u \right)^2}{N_0}, \quad (7)$$

and the system performance can be optimized by finding the optimal β .

III. MARKOV MODEL OF RELAY BATTERIES

To analyze the performance of the proposed ETMRS scheme, in this section we characterize the dynamic charging/discharging behaviors of the relay batteries. We follow [23], [25] and consider a discrete-level and finite-capacity battery model. Thus, it is natural to use a finite-state Markov chain (MC) to model the dynamic behaviors of relays' batteries. Recall that the proposed ETMRS scheme is a decentralized relay selection approach and the relays make decisions of their operation modes based only on their local CSI and battery status. Thus, we can evaluate the steady state distributions of all relays' batteries separately as they are independent to each other.

Let C denote the capacity of all relays' batteries and L denote the number of discrete levels excluding the empty level in each battery. Then, the i -th energy level of the relay battery can be expressed as $\varepsilon_i = iC/L$, $i \in \{0, 1, 2 \dots L\}$. It is worth pointing out that as shown in [32], the adopted discrete battery model can tightly approximate its continuous counterpart when the number of energy levels (i.e., L) is sufficiently large. For each relay node, we define state S_i as the relay residual energy in the battery being ε_i . The transition probability $T_u^{i,j}$ is defined as the probability of transition from state S_i to state S_j at the u -th relay. With the adopted discrete-level battery model, the amount of harvested energy can only be one of the discrete energy level. Thus, the discretized amount of harvested energy at the u -th relay during an EH operation is defined as

$$E_u \triangleq \varepsilon_j, \quad \text{where } j = \arg \max_{i \in \{0, 1, \dots, L\}} \left\{ \varepsilon_i : \varepsilon_i \leq \tilde{E}_u \right\}. \quad (8)$$

In the IF operation mode, based on the discrete-level battery model, the energy thresholds for each relay should be chosen from one of the energy levels of the battery excluding the empty level. The discretization of energy threshold for the u -th relay can be defined as

$$\beta_u \in \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_L\}. \quad (9)$$

We are now ready to describe the relay behaviors in the proposed ETMRS scheme mathematically. Let $\zeta_u[m] \in \{\zeta_E, \zeta_I\}$, $m = 1, 2, 3 \dots$, denote the operation mode of the u -th relay during the m -th transmission block, where ζ_E and ζ_I denote the EH and IF modes, respectively. We denote by $r_u[m]$ the residual energy accumulated by the u -th relay at the beginning of the m -th transmission block. Let φ_u represent the decoding indicator at the u -th relay for the first hop, which equals to 1 if the decoding is successful and 0 otherwise. According to the principle of the proposed ETMRS scheme, we then readily have the following equations

$$\zeta_u[m] = \begin{cases} \zeta_E, & \text{if } r_u[m] < \beta_u \\ \zeta_I, & \text{if } r_u[m] \geq \beta_u \end{cases}, \quad (10)$$

$$r_u[m+1] = \begin{cases} \min \left\{ r_u[m] + E_u, C \right\}, & \text{if } \zeta_u[m] = \zeta_E \\ r_u[m], & \text{if } \{\zeta_u[m] = \zeta_I\} \cap \{\varphi_u = 0\} \\ r_u[m] - \beta_u, & \text{if } \{\zeta_u[m] = \zeta_I\} \cap \{\varphi_u = 1\} \end{cases}. \quad (11)$$

Note that in (11), we consider that the energy consumption at the relays are dominated by their information forwarding operation and ignore other types of energy consumption (e.g., signal processing) for simplicity.

With the mathematical description of the proposed ETMRS scheme and the MC model defined above, we are now ready to evaluate the state transition probabilities of the MC for each relay node R_u , $u \in \{1, 2, \dots, N\}$. We summarize all the transition probabilities into the following four general cases.

1) *The battery remains unchanged (S_i to S_i with $0 \leq i \leq L$):* We can deduce that this transition case happens when EH operation is performed and the discretized amount of harvested energy is zero (i.e. $E_u = 0$), or IF operation is activated but the decoding of source's information at the relay is incorrect (i.e. $\varphi_u = 0$). The transition probability is thus given by

$$\begin{aligned} T_u^{i,i} &= \Pr \left\{ \{\zeta_u[m] = \zeta_E, E_u = 0 | r_u[m] = \varepsilon_i\} \cup \{\zeta_u[m] = \zeta_I, \varphi_u = 0 | r_u[m] = \varepsilon_i\} \right\} \\ &= \begin{cases} \Pr \{E_u = 0\}, & \text{if } \varepsilon_i < \beta_u \\ \Pr \{\varphi_u = 0\}, & \text{if } \varepsilon_i \geq \beta_u \end{cases}. \end{aligned} \quad (12)$$

We first evaluate the probability of zero harvested energy, $\Pr \{E_u = 0\}$. From the definition of discretization given in (8), condition $\tilde{E}_u = \frac{1}{2}\eta PH_u < \varepsilon_1 = C/L$ must hold such that the amount of harvested energy can be discretized to zero. Recall that the channels between source and relays are assumed to suffer from Nakagami-m fading. In this case, the PDF and CDF of H_u are given by [33, eq.2.21]

$$f_{H_u}(x) = \frac{b_u^{m_u}}{\Gamma(m_u)} x^{m_u-1} \exp(-b_u x), \quad (13)$$

$$F_{H_u}(x) = \frac{\gamma(m_u, b_u x)}{\Gamma(m_u)}, \quad (14)$$

where $b_u = m_u/\lambda_{SR_u}$. With the CDF, the probability of zero harvested energy is given by

$$\Pr \{E_u = 0\} = F_{H_u} \left(\frac{2C}{\eta PL} \right). \quad (15)$$

We now analyze the outage probability at the u -th relay. Let $\gamma_u = \frac{PH_u}{N_0}$ denote the received SNR at relay R_u . The channel capacity of the S - R_u link is given by

$$\Theta_u = \frac{1}{2} \log_2 (1 + \gamma_u). \quad (16)$$

According to the channel capacity, the term $\Pr\{\varphi_u = 0\}$ can be evaluated as

$$\Pr\{\varphi_u = 0\} = \Pr\{\Theta_u < \mathbb{R}\} = F_{H_u}\left(\frac{vN_0}{P}\right), \quad (17)$$

where \mathbb{R} is the system transmission rate and $v = 2^{2\mathbb{R}} - 1$ is the SNR threshold for system outage.

Based on the above analysis, the transition probability for this case can now be summarized as

$$T_u^{i,i} = \begin{cases} F_{H_u}\left(\frac{2C}{\eta PL}\right), & \text{if } \varepsilon_i < \beta_u \\ F_{H_u}\left(\frac{vN_0}{P}\right), & \text{if } \varepsilon_i \geq \beta_u \end{cases}. \quad (18)$$

2) *The non-full battery is fully charged (S_i to S_L with $0 \leq i < L$):* In this transition case, EH operation is performed to fully charge the battery. The amount of the harvested energy during this operation should be greater or equal to ε_{L-i} . Thus the transition probability can be characterized as

$$\begin{aligned} T_u^{i,L} &= \Pr\left\{\zeta_u[m] = \zeta_E, E_u \geq \varepsilon_{L-i} \mid r_u[m] = \varepsilon_i\right\} \\ &= \begin{cases} 1 - F_{H_u}\left(\frac{2(L-i)C}{\eta PL}\right), & \text{if } \varepsilon_i < \beta_u \\ 0, & \text{if } \varepsilon_i \geq \beta_u \end{cases}. \end{aligned} \quad (19)$$

3) *The battery is partially charged (S_i to S_j with $0 \leq i < j < L$):* Firstly, EH operation should be chosen to perform with the residual energy being level i . Secondly, the discretized amount of energy harvested during this EH operation should be exactly equal to $\varepsilon_{j-i} = (j-i)C/L$ and the transition probability is then given by

$$\begin{aligned} T_u^{i,j} &= \Pr\left\{\zeta_u[m] = \zeta_E, E_u = \varepsilon_{j-i} \mid r_u[m] = \varepsilon_i\right\} \\ &= \begin{cases} F_{H_u}\left(\frac{2(j-i+1)C}{\eta PL}\right) - F_{H_u}\left(\frac{2(j-i)C}{\eta PL}\right), & \text{if } \varepsilon_i < \beta_u \\ 0, & \text{if } \varepsilon_i \geq \beta_u \end{cases}. \end{aligned} \quad (20)$$

4) *The non-empty battery is discharged (S_j to S_i with $0 \leq i < j \leq L$):* Since the stored energy is reduced during the transition, IF operation should be invoked and the relay decodes the source's information correctly. Besides, the discharged amount of energy (i.e., $(j-i)$ levels of energy) should be equal to the energy threshold at the relay to guarantee that the residual energy after the discharge being i levels. Thus the transition probability is non-zero only when

$\varepsilon_{j-i} = \beta_u$. The transition probability can be evaluated as follows

$$\begin{aligned} T_u^{j,i} &= \Pr \{ \zeta_u[m] = \zeta_I, \varepsilon_{j-i} = \beta_u | r_u[m] = \varepsilon_j \} \\ &= \begin{cases} 1 - F_{H_u} \left(\frac{vN_0}{P} \right), & \text{if } \varepsilon_{j-i} = \beta_u \\ 0, & \text{if } \varepsilon_{j-i} \neq \beta_u \end{cases}. \end{aligned} \quad (21)$$

We now define $\mathbf{Z}_u = (T_u^{i,j})$ to denote the $(L+1) \times (L+1)$ state transition matrix of the MC for each relay R_u , $u \in \{1, 2, \dots, N\}$. By using similar methods in [23], we can easily verify that the MC transition matrixes \mathbf{Z}_u derived from the above MC model is irreducible and row stochastic. Thus for each relay R_u , there must exist a unique stationary distribution $\boldsymbol{\pi}_u$ that satisfies the following equation

$$\boldsymbol{\pi}_u = (\pi_{u,0}, \pi_{u,1}, \dots, \pi_{u,L})^T = (\mathbf{Z}_u)^T \boldsymbol{\pi}_u, \quad (22)$$

where $\pi_{u,i}$, $i \in \{0, 1, \dots, L\}$, is the i -th component of $\boldsymbol{\pi}_u$ representing the stationary distribution of the i -th energy level at relay R_u . The battery stationary distribution of relay R_u can be solved from (22) and expressed as [23]

$$\boldsymbol{\pi}_u = \left((\mathbf{Z}_u)^T - \mathbf{I} + \mathbf{B} \right)^{-1} \mathbf{b}, \quad (23)$$

where $\mathbf{B}_{i,j} = 1, \forall i, j$ and $\mathbf{b} = (1, 1, \dots, 1)^T$.

Moreover, when it comes to the i.i.d. channel model, all relays are equipped with an identical energy threshold and they have the same transition matrix, denoted by \mathbf{Z} . The corresponding identical stationary distribution of all relays can be similarly obtained as (23) and given by

$$\boldsymbol{\pi}_1 = \boldsymbol{\pi}_2 = \dots = \boldsymbol{\pi}_N = \boldsymbol{\pi} = (\pi_0, \pi_1, \dots, \pi_L)^T = \left((\mathbf{Z})^T - \mathbf{I} + \mathbf{B} \right)^{-1} \mathbf{b}. \quad (24)$$

IV. SYSTEM PERFORMANCE ANALYSIS AND OPTIMIZATION

Based on the stationery discrete distribution of relay batteries derived in Sec. III, in this section we first analyze the system outage probability of the proposed ETMRS scheme. Based on the derived closed-form expression, we subsequently discuss how to optimize the energy threshold of the relays to minimize the system outage probability.

A. Outage Probability Analysis

Let O denote the outage event of the considered system employing the proposed ETMRS scheme. According to the full probability theory, we can express the system outage probability as

$$P_{\text{out}} = \Pr\{O\} = \sum_{\Phi \in \Lambda} \Pr\{\Phi\} \Pr\{O|\Phi\}, \quad (25)$$

where $\Lambda = \{R_1, R_2, \dots, R_N\}$ denotes the set of all relays in the considered network and incorporates the decoding set Φ as its subset, $\Pr\{\Phi\}$ denotes the probability that the current decoding set is Φ , and $\Pr\{O|\Phi\}$ denotes the probability that system outage occurs under the decoding set Φ . In order to further expand (25), we define $\Phi_{k,n}$ to denote the n -th k -subset of Λ (i.e., the n -th k -subset of Λ contains exactly k elements, $k = 1, 2, \dots, N$, $n = 1, 2, \dots, \binom{N}{k}$). Then the outage probability of the ETMRS scheme can be further expanded as

$$P_{\text{out}} = \Pr\{\emptyset\} + \sum_{k=1}^N \sum_{n=1}^{\binom{N}{k}} \Pr\{\Phi_{k,n}\} \Pr\{O|\Phi_{k,n}\}, \quad (26)$$

by realizing that the system outage probability equals to one when the decoding set is an empty set \emptyset . The empty decoding set can be caused by two kinds of event: One is that none of the relays working in IF mode decodes the received signal from source correctly. The other is that all relays operate in EH mode and no relay performs IF. Based on the derived stationary discrete distribution of relay batteries given in (23), we can calculate the first probability term in (26) as follows

$$\begin{aligned} \Pr\{\emptyset\} &= \prod_{u: R_u \in \Lambda} \Pr\left\{\{\zeta_u = \zeta_I, \varphi_u = 0\} \cup \zeta_u = \zeta_E\right\} \\ &= \prod_{u: R_u \in \Lambda} \left(\Pr\{\varphi_u = 0\} \sum_{i=\beta_u/\varepsilon_1}^L \pi_{u,i} + \sum_{i=0}^{\beta_u/\varepsilon_1} \pi_{u,i} \right). \end{aligned} \quad (27)$$

Similarly, the term $\Pr\{\Phi_{k,n}\}$ can be computed as

$$\begin{aligned} &\Pr\{\Phi_{k,n}\} \\ &= \prod_{u: R_u \in \Phi_{k,n}} \Pr\{\zeta_u = \zeta_I, \varphi_u = 1\} \prod_{u: R_u \notin \Phi_{k,n}} \Pr\left\{\{\zeta_u = \zeta_I, \varphi_u = 0\} \cup \zeta_u = \zeta_E\right\} \\ &= \prod_{u: R_u \in \Phi_{k,n}} \left[(1 - \Pr\{\varphi_u = 0\}) \sum_{i=\beta_u/\varepsilon_1}^L \pi_{u,i} \right] \prod_{u: R_u \notin \Phi_{k,n}} \left(\Pr\{\varphi_u = 0\} \sum_{i=\beta_u/\varepsilon_1}^L \pi_{u,i} + \sum_{i=0}^{\beta_u/\varepsilon_1} \pi_{u,i} \right). \end{aligned} \quad (28)$$

To evaluate the third probability term in (26), we first characterize the distribution of the conditional end-to-end SNR for a given decoding set. For notation simplicity, we use $\gamma_{k,n}$ to denote the received SNR at the destination when the decoding set is $\Phi_{n,k}$. Recall that the conditional end-to-end SNR for a certain decoding set is given in (6), which includes a weighted sum of Rayleigh random variables. However, to the best knowledge of authors, the exact distribution of a weighted sum of Rayleigh random variables does not exist in open literature. As a result, we cannot further characterize the exact distribution of $\gamma_{k,n}$. Fortunately, with the aid of a tight approximation for the CDF of a weighted sum of Rayleigh random variables derived in [34], the CDF of $\gamma_{k,n}$ can be approximated as a gamma distribution and expressed as

$$F_{\gamma_{k,n}}(x) \approx \frac{\gamma\left(k, \frac{N_0}{4 \sum_{u: R_u \in \Phi_{k,n}} \beta_u \sigma_u^2} x\right)}{\Gamma(k)}, \quad (29)$$

where $\sigma_u = \sqrt{\lambda_{R_u D}/2}$ is the scale parameter of the Rayleigh fading channel between R_u and D .

In order to further expand the summation term in (29), we use a similar method adopted in [35]–[37]. To this end, we define a set $A = \left\{ \sqrt{\beta_u} \sigma_u : R_u \in \Lambda, u \in \{1, 2, \dots, N\} \right\}$ with the same cardinality as the set Λ (i.e., $|A| = |\Lambda|$). Similarly, let $A_{k,n}$ denote the n -th k -subset of A . The j -th element of the subset $A_{k,n}$ are denoted by $\phi_{k,n,j} \in A_{k,n}$, $j = 1, 2, \dots, k$. To be more clear, we list the corresponding relationship between $\phi_{k,n,j}$ and $\sqrt{\beta_u} \sigma_u$ in Table I.

With the corresponding relation between $\phi_{k,n,j}$ and $\sqrt{\beta_u} \sigma_u$, (29) can now be expressed as

$$F_{\gamma_{k,n}}(x) \approx \frac{\gamma(k, a_{k,n} x)}{\Gamma(k)} = 1 - \exp(-a_{k,n} x) \sum_{i=0}^{k-1} \frac{(a_{k,n} x)^i}{i!}, \quad (30)$$

where $a_{k,n} = \frac{N_0}{4 \sum_{j=1}^k \phi_{k,n,j}^2}$ and the last equality in (30) holds according to [28, Eq. (8.352.6)] with integer k . Based on the distribution of $\gamma_{k,n}$, the third probability term in (26) can now be further expanded as

$$\Pr\{O | \Phi_{k,n}\} = \Pr\{\gamma_{k,n} < v\} = F_{\gamma_{k,n}}(v) \approx 1 - \exp(-a_{k,n} v) \sum_{i=0}^{k-1} \frac{(a_{k,n} v)^i}{i!}. \quad (31)$$

By substituting (27), (28) and (31) into (26), we have derived an approximate closed-form expression of the outage probability for the proposed ETMRS scheme.

In terms of the i.i.d channel fading case where the relays have the same energy threshold, the number of different decoding sets reduces to $N + 1$ and the expression of system outage

k/n	n=1	n=2	...	n= $\binom{N}{k}$
k=1	$\phi_{1,1,1} = \sqrt{\beta_1}\sigma_1$	$\phi_{1,2,1} = \sqrt{\beta_2}\sigma_2$...	$\phi_{1,N,1} = \sqrt{\beta_N}\sigma_N$
k=2	$\phi_{2,1,1} = \sqrt{\beta_1}\sigma_1$ $\phi_{2,1,2} = \sqrt{\beta_2}\sigma_2$	$\phi_{2,2,1} = \sqrt{\beta_1}\sigma_1$ $\phi_{2,2,2} = \sqrt{\beta_3}\sigma_3$...	$\phi_{2, \frac{N(N-1)}{2}, 1} = \sqrt{\beta_{N-1}}\sigma_{N-1}$ $\phi_{2, \frac{N(N-1)}{2}, 2} = \sqrt{\beta_N}\sigma_N$
...
k=N-1	$\phi_{N-1,1,1} = \sqrt{\beta_1}\sigma_1$ $\phi_{N-1,1,2} = \sqrt{\beta_2}\sigma_2$... $\phi_{N-1,1,N-1} = \sqrt{\beta_{N-1}}\sigma_{N-1}$	$\phi_{N-1,2,1} = \sqrt{\beta_1}\sigma_1$ $\phi_{N-1,2,2} = \sqrt{\beta_2}\sigma_2$... $\phi_{N-1,2,N-2} = \sqrt{\beta_{N-2}}\sigma_{N-2}$ $\phi_{N-1,2,N-1} = \sqrt{\beta_N}\sigma_N$...	$\phi_{N-1,N,1} = \sqrt{\beta_2}\sigma_2$ $\phi_{N-1,N,2} = \sqrt{\beta_3}\sigma_3$... $\phi_{N-1,N,N-2} = \sqrt{\beta_{N-1}}\sigma_{N-1}$ $\phi_{N-1,N,N-1} = \sqrt{\beta_N}\sigma_N$
k=N	$\phi_{N,1,1} = \sqrt{\beta_1}\sigma_1$ $\phi_{N,1,2} = \sqrt{\beta_2}\sigma_2$... $\phi_{N,1,N-1} = \sqrt{\beta_N}\sigma_N$	—	...	—

TABLE I

THE RELATION BETWEEN $\phi_{k,n,j}$ AND $\sqrt{\beta_u}\sigma_u$, $u = 1, 2, \dots, N$, FOR $k = 1, \dots, N$, $n = 1, 2, \dots, \binom{N}{k}$ AND $j = 1, 2, \dots, k$.

probability can be simplified to

$$\hat{P}_{\text{out}} = \Pr\{\mathcal{O}\} = \Pr\{\hat{\emptyset}\} + \sum_{k=1}^N \Pr\{\hat{\Phi}_k\} \Pr\{\mathcal{O} | \hat{\Phi}_k\}, \quad (32)$$

where $\Pr\{\hat{\emptyset}\}$ is the probability that the decoding set is empty for the i.i.d. case, $\Pr\{\hat{\Phi}_k\}$ is the probability that the decoding set contains k relays, and $\Pr\{\mathcal{O} | \hat{\Phi}_k\}$ is the conditional outage probability when k relays falls in the decoding set. Similar to the analysis of the general case, the first and second probability terms in (32) can be expressed as

$$\Pr\{\hat{\emptyset}\} = \left(\Pr\{\varphi = 0\} \sum_{i=\beta/\varepsilon_1}^L \pi_i + \sum_{i=0}^{\beta/\varepsilon_1} \pi_i \right)^N, \quad (33)$$

$$\Pr\{\hat{\Phi}_k\} = \binom{N}{k} \left[(1 - \Pr\{\varphi = 0\}) \sum_{i=\beta/\varepsilon_1}^L \pi_i \right]^k \left(\Pr\{\varphi = 0\} \sum_{i=\beta/\varepsilon_1}^L \pi_i + \sum_{i=0}^{\beta/\varepsilon_1} \pi_i \right)^{N-k}, \quad (34)$$

where $\Pr\{\varphi = 0\}$ is the outage probability of the relays for the special i.i.d. channel fading case, which can be derived from that for the general case given in (17) and written as

$$\Pr\{\varphi = 0\} = \frac{\gamma\left(m, \frac{mvN_0}{\lambda_{SR}P}\right)}{\Gamma(m)}, \quad (35)$$

where m is the identical severity parameter of the Nakagami-m fading channels between source and relays and λ_{SR} is the identical channel power gain of the first hop.

The third probability term in (32) can be deduced from (31) as

$$\Pr\left\{O \mid \widehat{\Phi}_k\right\} \approx 1 - \exp\left(-\frac{N_0v}{4k\beta\sigma^2}\right) \sum_{i=0}^{k-1} \frac{\left(\frac{N_0v}{4k\beta\sigma^2}\right)^i}{i!}, \quad (36)$$

where $\sigma = \sqrt{\lambda_{RD}/2}$ is the identical scale parameter for the Rayleigh fading channels between relays and destination. Note that different from (31), the expression of the conditional system outage probability given in (36) does not require the parameters defined in Table I. The outage probability for the special i.i.d. channel fading case can be obtained by substituting (33), (34) and (36) into (32).

B. Energy Threshold Optimization

We first consider the special i.i.d. channel fading case, when the energy threshold of the relays β grows, the conditional outage probability derived in (36) reduces and the overall system outage probability correspondingly decreases. On the other hand, according to (34), increasing the energy threshold β will decrease the probability of each relay working in IF mode and thus the number of relays falling into the decoding set. This will increase the overall system outage probability. In simple words, when the designed energy threshold β for the relays is small, most of relays could fall into the decoding set but their transmit power is low. When the energy threshold is large, only a few relays fall into the decoding set but their transmit power is high. We can now infer that for the special i.i.d. channel fading case, there must exist an optimal energy threshold $\beta \in \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_L\}$ that minimize the system outage probability of the proposed ETMRS scheme. Similarly, for the general i.n.i.d. channel fading case where the relays may be located quite differently. There should also exist an optimal energy threshold set $\{\beta_1, \beta_2, \dots, \beta_N\}, \beta_u \in \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_L\}, \forall u = 1, 2, \dots, N$, for all relays that minimize the overall system outage probability.

Due to the complexity of the adopted MC model, it is difficult to derive a closed-form expression for the optimal energy threshold of each relay. However, for the special i.i.d. channel fading case, the optimal energy threshold can be easily achieved by performing a one-dimension exhaustive search from all the possible energy levels with the derived analytical outage probability given in (32). The computation complexity of this search is given by $O(L)$.

When it comes to the general i.n.i.d. case, the optimal set of energy thresholds can be found by a N -dimension exhaustive search from all the possible combinations of energy thresholds with the analytical expression derived in (26). The computation complexity is $O(L^N)$, which grows exponentially with the number of relays N . Thus, finding the optimal set of energy thresholds becomes computationally prohibitive when the number of relays N and the level of batteries L are large. To overcome this problem, in the following subsection, we provide a heuristic approach to design the energy thresholds for the general i.n.i.d. case.

C. A Heuristic Approach for i.n.i.d Scenario

Intuitively, a higher energy threshold should be set for those relays closer to the source node. This is because that the average amount of harvested energy is higher for those relays and they can support a higher transmit power to reduce the overall system outage probability. On the other hand, relays closer to the source node are relatively farer away from the destination node such that the associated second hop channels are relatively weaker, a higher energy threshold should also be used to overcome the larger path loss. Similarly, for the relays near the destination node, smaller energy thresholds can be adopted due to their limited harvested energy and stronger second hop channels. Inspired by this fact, we design the energy threshold for each relay R_u in the considered network as

$$\tilde{\beta}_u = z\lambda_{SR_u}/\lambda_{R_uD}, \quad (37)$$

where z is a scalar factor to adjust the overall transmit power of all the relays. For the considered discrete-battery model, the designed energy threshold $\tilde{\beta}_u$ must be discretized to one specific energy level of the battery. We define the discretized energy threshold for the u -th relay as

$$\beta_u \triangleq \varepsilon_j, \quad \text{where } j = \begin{cases} \arg \min_{i \in \{0,1,\dots,L\}} \left\{ \varepsilon_i : \varepsilon_i \geq \tilde{\beta}_u \right\}, & \text{if } \tilde{\beta}_u \leq \varepsilon_L \\ L, & \text{if } \tilde{\beta}_u > \varepsilon_L \end{cases}. \quad (38)$$

Based on the above definitions, the discretized energy threshold for relay R_u can be further expressed as

$$\beta_u = \min \left(\varepsilon_1 \left\lceil \frac{z \lambda_{SR_u}}{\lambda_{R_u D} \varepsilon_1} \right\rceil, C \right). \quad (39)$$

Similar to the analysis given in the previous subsection, we can deduce that there should exist an optimal value of z that minimize the system outage probability. To find the optimal value of z , we first define λ_{\max} and λ_{\min} as the maximum and minimum value of the term $\lambda_{SR_u}/\lambda_{R_u D}$, among all relays, respectively. The optimal z should exist within the interval $[\varepsilon_1/\lambda_{\max}, \varepsilon_L/\lambda_{\min}]$, where $z = \varepsilon_1/\lambda_{\max}$ could make all relays choose ε_1 as their energy thresholds and $z = \varepsilon_L/\lambda_{\min}$ will force all relays to adopt energy level L as their energy thresholds. The optimal z thus can now be achieved by performing a one-dimension exhaustive search from this interval over the derived outage probability expression. In order to capture all the possible combinations, we search z with an increment of $\varepsilon_1/\lambda_{\max}$ each time and the computation complexity for the proposed heuristic approach can be expressed by $O(L\lambda_{\max}/\lambda_{\min})$. Note that the complexity of the heuristic approach is obviously much lower than that of the exhaustive search given by $O(L^N)$.

V. PERFORMANCE UPPER BOUND

As described in Section III, in this paper we adopt a practical finite-capacity battery model. With this model, it can be readily deduced that the system performance could be improved when we increase the battery capacity (i.e., C). This is because a larger battery capacity can reduce the energy loss caused by energy overflow (i.e., the battery cannot be charged when it is full) and thus the relays have more available energy to support their information forwarding operation. On the other hand, we can infer that the system performance improvement speed actually decreases as the battery capacity increases since the energy overflow happens more rarely when the battery capacity keeps increasing. A natural question that comes up here is “*For a given network setup, how large of the battery capacity C will be sufficient?*” This question is particularly important for the considered system as one of its potential applications will be low-cost and lower-power networks (e.g., wireless sensor networks), in which the network deployment cost should be kept as low as possible by carefully selecting the battery capacity. However, it is hard to find the answer of this question based on the derived expressions in the previous section due to their

complexity. We are thus motivated to adopt an indirect way: We first derive performance upper bounds of the considered system and the *sufficient* battery capacity will then be obtained when a certain value of C can make the performance expressions derived in previous sections approach their corresponding upper bounds. In this sense, in this section we analyze the performance upper bounds of the considered system with infinite battery capacity (i.e., $C \rightarrow \infty$ and implicitly $L \rightarrow \infty$).

When the relay batteries are infinite, there will be no energy overflow. As such, we can implement the well-known flow conservation law to evaluate the system outage probability. Specifically, we can treat the total amount of harvested energy in the long term equal to the total amount of energy consumed for information forwarding. Mathematically, we have the following formula for the u -th relay

$$p_u \bar{E}_u = q_u \beta_u, \quad (40)$$

where p_u denotes the probability that relay R_u performs EH operation, q_u denotes the probability that the u -th relay falls in the decoding set, \bar{E}_u is the average amount of harvested energy during each EH operation and β_u is the energy threshold of R_u adopted in the finite-capacity case. Since both C and L are infinity, the term \bar{E}_u can be obtained from (2) given by

$$\bar{E}_u = \frac{1}{2} \eta P \lambda_{SR_u}. \quad (41)$$

With the fact that relay R_u falls in the decoding set only when it performs IF operation and decodes the source's information correctly, we thus have

$$q_u = (1 - p_u) (1 - \Pr \{\varphi_u = 0\}), \quad (42)$$

where the term $\Pr \{\varphi_u = 0\}$ has been derived in (17). By jointly considering (40)-(42), we can now obtain q_u expressed as

$$q_u = \frac{1}{\frac{1}{1 - \Pr \{\varphi_u = 0\}} + \frac{2\beta_u}{\eta P \lambda_{SR_u}}}. \quad (43)$$

The first and second probability terms in the system outage probability defined in (26) can now be evaluated by q_u as

$$\Pr \{\emptyset\} = \prod_{u: R_u \in \Lambda} (1 - q_u), \quad (44)$$

$$\Pr \{\Phi_{k,n}\} = \prod_{u: R_u \in \Phi_{k,n}} q_u \prod_{u: R_u \notin \Phi_{k,n}} (1 - q_u). \quad (45)$$

The third probability term in (26) is independent of C and L , and only depends on the energy threshold of the relays. Note that we should adopt the same energy thresholds as the practical finite-capacity case to achieve a meaningful upper bound. In this sense, the third probability term in (26) remains the same as the finite-capacity case, which has been derived in (31). We thus can express the upper bound of system outage probability for the proposed ETMRS scheme corresponding to infinite battery capacity as

$$P_{\text{out}}^{ub} \approx \prod_{u: R_u \in \Lambda} (1 - q_u) + \sum_{k=1}^N \sum_{n=1}^{\binom{N}{k}} \left[\prod_{u: R_u \in \Phi_{k,n}} q_u \prod_{u: R_u \notin \Phi_{k,n}} (1 - q_u) \right] \times \left[1 - \exp(-a_{k,n}v) \sum_{i=0}^{k-1} \frac{(a_{k,n}v)^i}{i!} \right]. \quad (46)$$

When the special i.i.d. channel fading case is considered, similar to the above analysis, the system outage probability is upper bounded by

$$\widehat{P}_{\text{out}}^{ub} \approx (1 - q)^N + \sum_{k=1}^N \binom{N}{k} q^k (1 - q)^{N-k} \left[1 - \exp\left(-\frac{N_0 v}{4k\beta\sigma^2}\right) \sum_{i=0}^{k-1} \frac{\left(\frac{N_0 v}{4k\beta\sigma^2}\right)^i}{i!} \right], \quad (47)$$

where q is the probability of each relay falling in the decoding set when the channels are i.i.d. and it can be easily derived based on (43) and expressed as

$$q = \frac{1}{\frac{1}{1 - \Pr\{\varphi=0\}} + \frac{2\beta}{\eta P \lambda_{SR}}}. \quad (48)$$

Remark 1: By substituting (17) into (43), we can see that the probability for the u -th relay falling in the decoding set (i.e., q_u) is proportional to the term $P\lambda_{SR_u}$. This is understandable since as the value of $P\lambda_{SR_u}$ increases, the u -th relay harvests more energy on average and also achieves lower outage probability in the first hop. Moreover, as expected, the value of q_u is inversely proportional to that of the energy threshold β_u . Furthermore, using the above performance upper bounds of system outage probability, we are able to judge whether a certain value of battery capacity C is sufficiently large for a given network setup via numerical results as shown in next section.

VI. NUMERICAL RESULTS

In this section, we present some numerical results to validate and illustrate the above theoretical analysis. In order to capture the effect of path-loss, we use the model that $\lambda_{XY} = \frac{1}{1+d_{XY}^\alpha}$, where

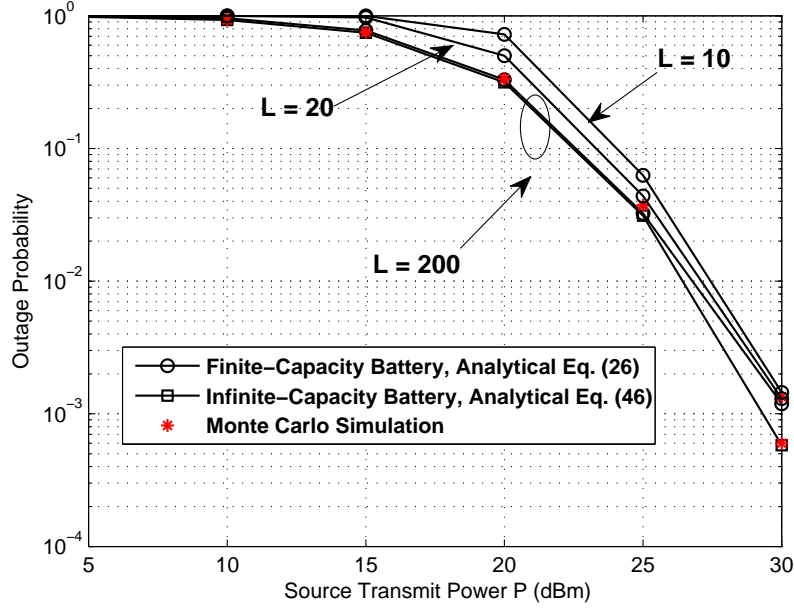


Fig. 3. The outage probability of the proposed ETMRS scheme versus the source transmit power for different battery levels L , where $R = 1$, $C = 0.01$, $\mathbf{d}_{SR} = \{5, 5.5, 6, 6, 6, 6, 6.5, 7\}$ and $\boldsymbol{\beta} = \{2, 2, 2, 2, 2, 2, 1, 1\} \times 10^{-3}$.

λ_{XY} is the average channel power gain between node X and Y , d_{XY} denotes the distance between node X and Y , and $\alpha \in [2, 5]$ is the path-loss factor. For simplicity, we consider a linear topology that the relays are located on a straight line between the source and destination and denote by $\mathbf{d}_{SR} = \{d_{SR_1}, d_{SR_2}, \dots, d_{SR_N}\}$ the set of distances between source and all relays. We use $\boldsymbol{\beta} = \{\beta_1, \beta_2, \dots, \beta_N\}$ to represent the energy threshold set for all the relays. In all the following simulations, we set the distance between the source and destination $d_{SD} = 20\text{m}$, the path-loss factor $\alpha = 2$, the severity parameter $m_u = 2, \forall u$, the noise power $N_0 = -80\text{dBm}$ and the energy conversion efficiency $\eta = 0.5$.

We first compare the analytical system outage probability with the Monte Carlo simulation, which corresponds to the practical case that the charging of the relay batteries is continuous (i.e., $L \rightarrow \infty$). To this end, we plot outage probability of the proposed ETMRS scheme versus the source transmit power for different battery levels L in Fig. 3. We can see that the derived analytical expression of outage probability approaches the corresponding Monte Carlo simulation result as the discrete battery level L increases. Specifically, when $L = 200$, the analytical expression coincides well with the simulation result, which verifies the effectiveness

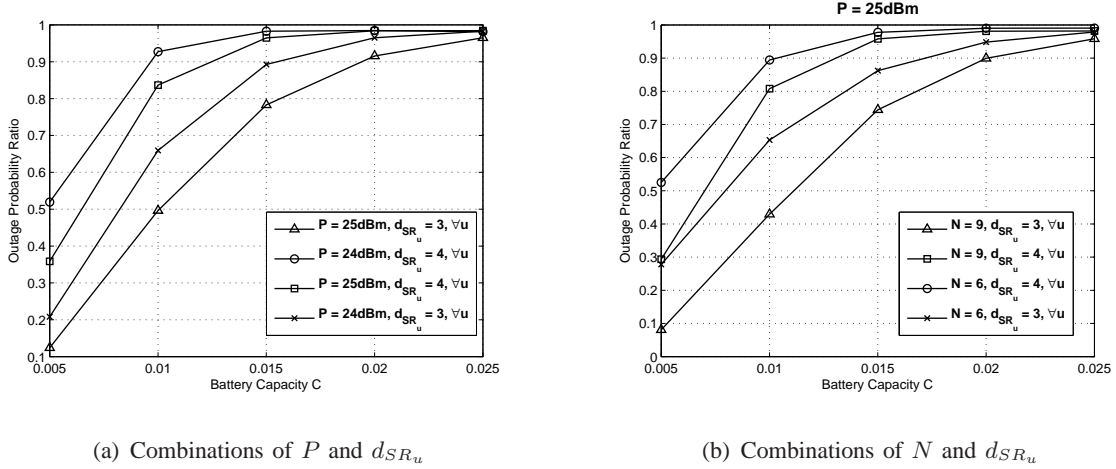


Fig. 4. The ratio between the upper bound outage probability of the ETMRS scheme with infinite battery capacity and the outage probability with finite battery capacity versus battery capacity C for different source transmit power and relay topologies, where $R = 1$, $\beta_u = 0.001$, $\forall u$ and $L = 600$.

of the adopted MC model and the correctness of our theoretical analysis presented in Sec. II-IV. Furthermore, it can be observed that the accuracy of the MC model improves as source transmit power increases. This is because that for the same degree of precision, smaller interval between adjacent energy levels (i.e., larger L for a given C) is required to accurately characterize the charging amount of the battery when the harvested energy is low. Moreover, as expected, the performance of the ETMRS scheme with finite-capacity battery is bounded by the one with infinite-capacity battery. As the analytical results agree well with the simulation results and for the purpose of simplicity, in the following, we will only plot the analytical results of the ETMRS scheme.

Recall that we derived system performance upper bound in Section V, which can be used to judge whether a certain value of C is sufficiently large via numerical results. To show this, we now plot the outage probability ratio of the proposed ETMRS scheme with finite battery capacity over its upper bound with infinite battery capacity versus relay battery capacity C in Fig. 4. For simplicity, we consider the special i.i.d. case where the relays are located in a cluster. The outage probability ratio is formally defined as $\hat{P}_{\text{out}}^{ub}/\hat{P}_{\text{out}} \in (0, 1]$, where $\hat{P}_{\text{out}}^{ub}$ derived in (47) is the system outage probability for infinite-capacity battery case and \hat{P}_{out} given in (32) is the system outage probability for finite-capacity battery case. From Fig. 4, we can first observe that the outage probability ratio monotonically increases as the battery capacity C

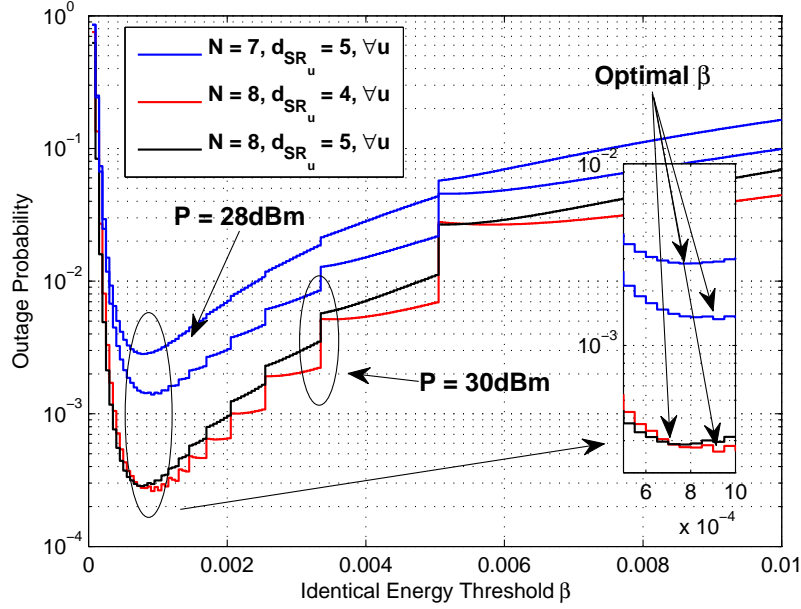


Fig. 5. The outage probability of the proposed ETMRS scheme versus the identical energy threshold for different source transmit power, relay topologies where $R = 1$, $C = 0.01$ and $L = 200$.

grows and gradually converges to 1 when the value of C is large enough. This indicates that the performance gap between the finite-capacity battery and infinite-capacity battery decays to 0 as C increases. However, the convergence speed varies for different network setups. Specifically, from Fig. 4(a), we can see that the outage probability ratio converges to 1 slower with either higher source transmit power or shorter distances between source and relays. This can be understood as follows. The amount of harvested energy at relays increases when the source transmit power increases or the distances between source and relays reduce. In this case, a larger battery capacity is required to avoid the potential energy overflow (i.e., the battery cannot be charged when it is full). From Fig. 4(b), we find out that as the number of relays N increases, the convergence speed is also reduced. This is understandable since energy overflow happens in a higher probability as N grows. Similarly, larger value of C is required to make the outage probability ratio close to 1. We can now summarize that larger capacity batteries should be equipped at energy harvesting relays for those network setups with higher source transmit power, shorter distances between source and relays or larger number of relays.

In Fig. 5, we plot the outage probability versus the energy threshold of the i.i.d. channel fading

case for different source transmit power and relay location. Recall that the energy threshold β is discretized to one of the $L + 1$ energy levels of the battery excluding the empty level. Thus, the outage probability is plotted in stair curve in the figure. First of all, Fig. 5 demonstrates that there exists an optimal energy threshold that minimizes the outage probability in all considered cases, which validates our deduction in Remark 1. Moreover, we can see from Fig. 5 that the higher the transmit power at the source, the larger the value of optimal energy threshold. This is because the relays can harvest more energy when the transmit power of the source increases and thus a larger energy threshold can be supported. We can also see from Fig. 5 that a smaller energy threshold should be chosen when the relays are located away from the source. This can be explained as two folds. Firstly, the harvested energy at the relays is limited when they are far away from the source. Secondly, the second hop channel condition becomes better when the relays are away from the source (i.e., close to the destination) and a small energy threshold is enough to avoid system outage. The number of the relays N also affects the optimal energy threshold. From Fig. 5 we can see that as the number N increases, the optimal energy threshold decreases. This is understandable since in cooperative relay networks, a decoding set with more relays can achieve the same outage probability with less transmit power. Note that similar results can be observed for the general i.n.i.d case, which are not provided here due to space limitation.

We then compare the system performance with optimized energy threshold using the exhaustive search based approach and the proposed heuristic one. As the optimal energy threshold for the i.i.d. case can be easily obtained via a one-dimension exhaustive search, we only consider the general i.n.i.d. case where the optimal energy thresholds for the relays could be different. In Fig. 6, we plot the system outage probability for these two approaches versus the source transmit power for two different relay topologies. The outage probability of exhaustive search approach is obtained via a N -dimension exhaustive search from all the possible combinations of energy thresholds. Due to the intense computation complexity mentioned before, we consider the case that each relay only has 20 energy levels. For the proposed heuristic approach, the computation complexity can be dramatically reduced and it is thus particularly suitable to those networks with large number of relays and energy levels. From Fig. 6, we can observe that the proposed heuristic approach with reduced complexity can achieve the near optimal system performance in both simulated scenarios.

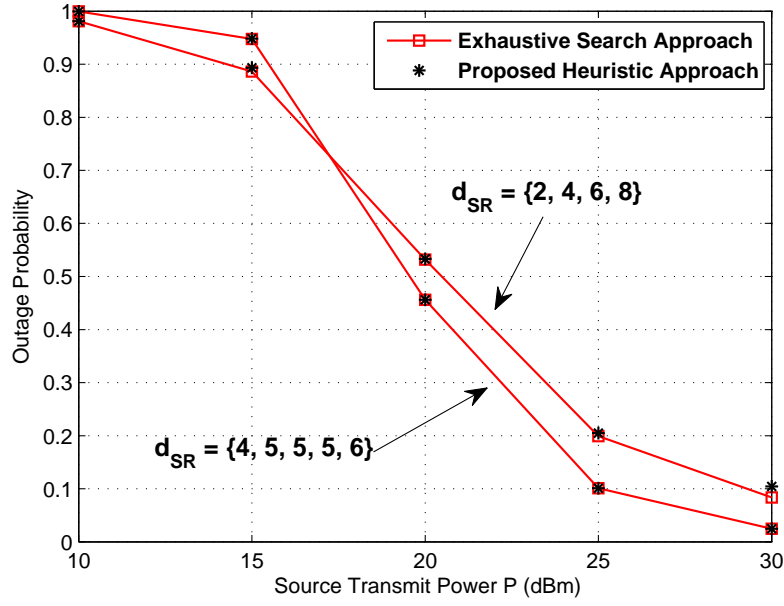


Fig. 6. Outage probabilities of the proposed ETMRS with optimized energy thresholds using exhaustive search approach and the proposed heuristic approach for different relay locations, where $C = 0.01$ and $L = 20$.

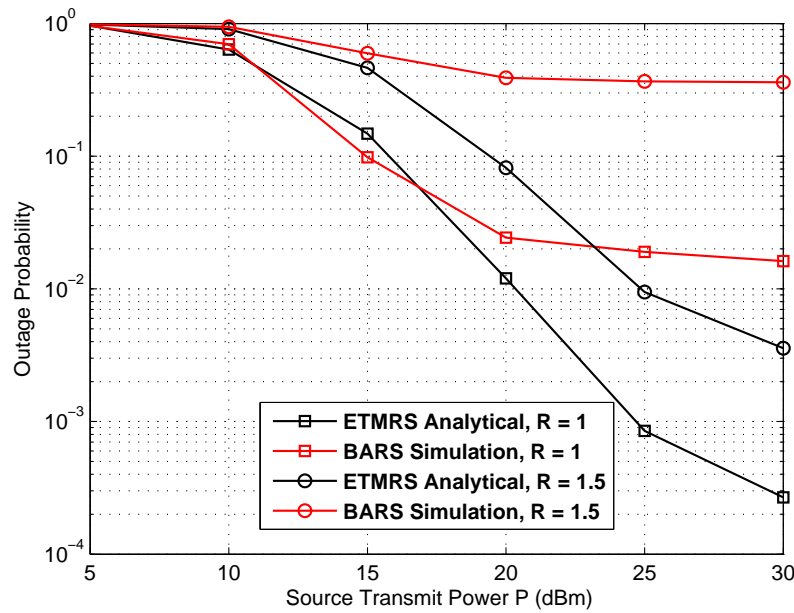


Fig. 7. The outage probability of the proposed ETMRS scheme and the existing BARS scheme with optimal settings versus source transmit power for different modulation methods, where $N = 8$, $d_{SR_u} = 3$, $\forall u$, $C = 0.01$ and $L = 200$.

Finally, we compare the proposed ETMRS scheme with the existing BARS scheme [25] in Fig. 7. As the BARS scheme proposed in [25] only considers a clustered topology, we thus compared BARS scheme with the special i.i.d. case of the proposed ETMRS scheme. In Fig. 7, we plot the optimal outage probabilities of the ETMRS and BARS schemes versus source transmit power for different transmission rate. It can be observed that the proposed ETMRS scheme can achieve a lower outage probability than the BARS scheme. And their performance gap is enlarged as the transmission rate grows.

VII. CONCLUSIONS

In this paper, we proposed an energy threshold based multi-relay selection (ETMRS) scheme for accumulate-then-forward energy harvesting relay networks. We modeled the finite-capacity battery of the relays by a finite-state Markov Chain (MC) in order to evaluate their stationary distribution. We then derived an approximate closed-form expression for system outage probability of the proposed ETMRS scheme over independent but not necessarily identical mixed Nakagami-m and Rayleigh fading channels. A heuristic approach was then designed to minimize the system outage probability, which was shown to achieve near-optimal performance with reduced computational complexity. Moreover, we derived upper bounds for the concerned system performance corresponding to the case that all relays are equipped with infinite-capacity batteries. Numerical simulations validated the accuracy of the analytical results, demonstrated the impact of various system parameters and provided some insights of practical relay battery design. Numerical results showed that larger capacity battery should be equipped at energy harvesting relays for those network setups with higher transmit power, shorter distance in the first hop or larger number of relays. Furthermore, the proposed ETMRS scheme can considerably outperform the existing single relay selection scheme and the performance gap between them is further enlarged as the system transmission rate increases.

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